Imp

* Identities Related to Regular Expressions:-

$\phi + r = r$

$\phi . r = r . \phi = \phi$

$\epsilon . r = r . \epsilon = r$

$\epsilon^* = \epsilon$ and $\phi^* = \epsilon$

$r + r = r$

$r^* . r^* = r^*$

$r . r^* = r^* . r = r^+$

$$\boxed{\begin{array}{l} r = \epsilon^+ = \{\epsilon\} \\ r^+ \cup r^* = r^* \\ r^* . r^+ = r^+ \end{array}}$$

$(r^*)^* = r^*$ $\quad$ $\boxed{(r^*)^+ = r^*}$

$\epsilon + r . r^* = r^* = \epsilon + r . r^*$

$(p . q)^* . p = p . (q . p)^*$

$(p+q)^* = (p^* . q^*)^* = (p^* + q^*)^*$

$(p+q) . r = p . r + q . r$ and $r . (p+q) = r . p + r . q$

$(a+b)^* \neq (a . b)^*$

Imp

* Arden's Theorem with Proof

If P and Q are two regular expressions over Σ, and if P does not contain ϵ, then the following equation in R given by, $R = Q + RP$ has a unique solution i.e. $R = QP^*$

Proof :- $R = Q + RP$ — (i)

Replace R with QP^* on R.H.S.

$$= Q + QP^*P$$

$$= Q(\epsilon + P^*P) \quad [\epsilon + R^*R = R^*)$$

P^*

$$= Q(P^*) \quad \text{or} \quad \boxed{R = QP^*}$$

Hence Proved

$R = Q + RP$ —— (ii)

Replace R with $Q + RP$

$$= Q + [Q + RP]P$$

$$= Q + QP + RP^2$$

$Q + RP$

$$= Q + QP + (Q + RP)P^2$$

$$= Q + QP + QP^2 + RP^3$$

⋮

$$Q + QP + QP^2 + \ldots\ldots QP^n + RP^{n+1} \text{ —— (iii)}$$

Replace again R with QP^*

$= Q + QP + QP^2 + \ldots\ldots QP^n + QP^* P^{n+1}$

$= Q\,[\underbrace{\epsilon + P + P^2 \ldots\ldots P^n + P^* P^{n+1}}_{\to\, P^*}]$

$\boxed{R = QP^*}$

↳ It is Proved that (QP^*).

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